Konformní transformace křivosti

$g_{ab} \to \tilde{g}_{ab} = \Omega^2 g_{ab}$

$\nabla \to \tilde{\nabla} = \nabla + Q$ $\qquad Q^a_{bc} = \lambda_b \delta^a_c + \lambda_c \delta^a_b - g^{am} \lambda_m g_{bc}$

$\lambda_a = d_a \ln \Omega = \Omega^{-1} d_a \Omega$

jak se transformuje

$\tilde{R}_{ab}{}^c{}_d \qquad \tilde{R}_{abcd}$

$\tilde{Ric}_{ab} \qquad \tilde{R}$ $\qquad ?$

$\tilde{C}$

kde $\quad C_{abcd} = R_{abcd} - \frac{4}{d-2} Ric_{[a][c} g_{d][b]} + \frac{2}{(d-1)(d-2)} R\, g_{[a][c} g_{d][b]}$

$$\tilde{R}_{abcd} = \Omega^2 \left[R_{abcd} - 4\, g_{[a][c} \nabla_{d]} \nabla_{b]} \ln \Omega - 4 \lambda_{[a} g_{b][c} \lambda_{d]} - 2 \lambda^2 g_{[a][c} g_{d][b]} \right]$$

$$\tilde{Ric}_{ab} = Ric_{ab} - (d-2) \nabla_a \nabla_b \ln \Omega - g_{ab} \Box \ln \Omega$$
$$+ (d-2) \lambda_a \lambda_b - (d-2) g_{ab} \lambda^2 \qquad\qquad \lambda^2 = g^{ab} \lambda_a \lambda_b$$

$$\tilde{R} = \Omega^{-2} \left[R - 2(d-1) \Box \ln \Omega - (d-1)(d-2) \lambda^2 \right]$$

$$\tilde{C}_{ab}{}^k{}_l = C_{ab}{}^k{}_l \qquad\qquad \tilde{C}_{abcd} = \Omega^2 C_{abcd}$$

$$\tilde{Cot}_{mab} = Cot_{mab} + C_{ab}{}^n{}_m \lambda_n$$

3D: $\quad C_{abcd} = 0 \qquad \tilde{Cot}_{mab} = Cot_{mab}$

Jak se transformuje

$$\tilde{\Box}\,\tilde{\phi}$$

kde $\tilde{\Box} = \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b$ $\qquad \tilde{\phi} = \Omega^s \phi$

$$\tilde{\Box}\tilde{\phi} = \Omega^{s-2}\Big[\,\Box\phi - (2s+d-2)\,\lambda\cdot d\phi - s(\Box \ln\Omega)\,\phi - s(s+d-2)\,\lambda^2\phi\,\Big]$$

$$\lambda\cdot d\phi = g^{ab}\lambda_a d_b\phi \qquad \Box = g^{ab}\nabla_a\nabla_b \qquad \lambda^2 = g^{ab}\lambda_a\lambda_b$$

$\Downarrow$

$$\tilde{\Box}\tilde{\phi} + \xi\tilde{R}\tilde{\phi} = \Omega^{-\left(\frac{d}{2}+1\right)}\left(\Box\phi + \xi R\phi\right)$$

pro $d>1$ $\qquad s = -\frac{d}{2}+1 \qquad \xi = \frac{1}{4}\,\frac{d-2}{d-1}$

$d=4 \qquad s=-1 \qquad \xi = \frac{1}{6}$